4. 3. Mathematical thinking

Content

4. 3. 1 A preliminary definition

That mathematics is applied logic is so obvious that we do not dwell on its argument. That mathematics in its present form - or rather wealth of forms - is "a logically coherent system of objective sentences" is not so immediately obvious.

1. Its stormy development means that a single person can hardly oversee its entirety.

2. The problem is the term "objective. Opinions differ in virtue of the metaphysics that shows itself in it. The nominalist will easily call them a construction of the mind that so to speak "hanging in the air" unless there are additional mathematical applications. The abstractionist sees them as their own form of reality in themselves while the ideative sees in them a realization of ideas. In all cases, the founders of logistics were essentially Platonicians.

Quantity: - Ch. Lahr, *Logique*, Paris, 1933-27, 559 / 569 (*Les sciences mathématiques*) states, "Mathematics is the science of quantity."

Lahr defines "quantity" as both number mathematical and space mathematical quantity. Note :Very briefly considering the huge number of mathematical equations that take as their basic form the differential "greater than / equal to / less than." Which is clearly quantile to be understood. For geometry or space mathematics, the quantitative is obvious in its own way.

A new definition. - P.J. Davis / R. Hersh, *l' Univers mathématique*, Paris, 1985, 6 says: a naive definition, in its place in the dictionary and suitable as a first approximation, reads, "Mathematics is the science of quantity and space."

1. The authors add, "...as well as of the symbol system that connects quantity and space."

2. They further state that a. that definition "rests on real historical grounds" and that they make it their starting point but then to b. depict the developments of mathematics since the last centuries and the different interpretations of mathematics in the broadening definition. - Remains that arithmetic (quantitative aspect) and geometry (spatial aspect) remain starting points for Davis and Hersh, for historical and practical reasons.

A substantive definition of mathematics in its present forms is then rather to be understood as some lemma, i.e., a provisional definition.

4. 3. 2 Mathematical and non-mathematical evidential power.

Bibl. St.: J. Chlebny, *les maths font leur preuves*, in Journal de Genève, Gazette de Lausanne 10/11.09.1994. - At the 22nd International Congress of Mathematics (Zurich), P.L. Lions (b. 1956) received the Fields honorary mark for his meritorious work in the field of applied mathematics.

The distinction between mathematical and non - mathematical proofs. - Here's how Lions puts it. - "If mathematicians are sometimes not very popular with some scientists, it is because of the thorough importance mathematicians attach to the proof.

1. ***Mathematical***. - "Mathematics is the only science that provides definitive and irrevocable proofs, based on a kind of reduction that leads to an indisputable result." Thus Chlebny.

2. ***Not mathematics***. - "The other subject sciences test a theory against some experience. These inevitably involve inaccuracies.

Applicative model. - According to physics, the fall of bodies is governed by a very simple law. Nevertheless, observation in itself is not proof. One has to take into account e.g. the friction in the air, the time the applied equipment needs to react. Thus the law, although theoretical, cannot be tested exactly. - So much for Clebny's report.

Note - Whether all physicists agree with this is questionable. However, it is a fact that non - mathematical testing (of a law, of a theory e.g.) is situational, i.e. occurs within a context of circumstances with the happening influences of others. Whereas mathematical proofs take place outside such situations, - put on paper in the pure mind.

Note - Ch. Lahr, *Logique*, Paris, 1933-27, 566/569 (*la démonstratrice*) says that the main types of reasoning in mathematics are the following.

1. Deductive. Axioms and propositions derived from those axioms serve as sufficient grounds for logically rigorous deduction of further conclusions from them.

2. Reductive.- One posits (as a lemma) a theorem as to be proved, and then step by step (algorithmically) provides the proof (as analysis).

Note: This is correct in an empirical mathematics but within an axiomatic - deductive system this second, so called reductive type amounts to a deductive proof based on the presupposed and 'foundational' axioms and theorems deduced from them. - One thinks of the so-called mathematical induction e.g..

4. 3. 3 Mathematical induction

Bibl. st.: W.St. Jevons, *Logic*, 168/171. We pause to consider what the author says.

Geometric induction. Euclid, Elements, 1: 5, states, "The angles at the base of an isosceles triangle are equal to each other." Note: they are each other's metaphorical or similarity model. Proof. One draws exactly one isosceles triangle. One shows that, if the sides are equal, then the opposite angles are necessarily equal. Remark: the opposite angles are metonymic or coherence models of the sides because, though they are not similar, they are related to them (and provide information about their sides, (cf. 6.9)). Euclid leaves it at this one sample. The one triangle is a paradigm so that in and through that unique model all possible models are summarized. That this is possible stands or falls with the absolute requirement - ceteris paribus - that it is about isosceles triangles. In other words: the summative induction here is limited to just one sample with the condition of isosceles triangles. Thus an amplificative induction is logically justified.

Number mathematical induction. Jevons gives a paradigm. Given: the two first consecutive odd numbers, 1 and 3. If added together, their sum is 1+3 = 4 = 2 x 2. Given: three such numbers, 1 + 3 + 5, whose sum is 9 = 3 x 3. Analogously: 1 + 3 + 5+ 7 = 16 = 4 x 4. One can already see the "rule"! This is a summative induction (three samples), summarizable in the statement "So far, the sum of all such (note our term 'such', which is similarity) numbers is equal to the second power of the number of numbers." Now amplificatory induction follows thanks to algebraization (letter numbers).

Given: n number of consecutive odd numbers, starting with 1.

Hypothesis: "The established law holds up to and including the nth term".

This gives: $1+3+5+7+ \ldots (2n-l) = n^2$.

This is now applied to the successor 2n+1: $1+3+5+7+ \ldots (2n-l) + (2n+l)$.

The sum of this last number with all previous ones is identic to $(n+l)^2$.

General decision: "If the law holds for n terms, then the law also holds for n+ 1 terms." One sees the term "general decision" in which "general" interprets knowledge-expanding induction.

Jevons' comment. The only difference with the geometric induction above is that the chosen cases are the first of the set of integers for reason of its orderliness. The smallness of the number of chosen tests is given emphasis. As summative inductions, they suffice on one condition, namely, that they provide logical certainty.

Note: Fundamentally, the deliberately chosen paradigms are haphazard paradigms whose surveyability elicits preference. But there is nothing more: since they represent a general "law," they are fundamentally haphazard because what is true for the chosen examples is true for any other sample. u, "induction" in one of its principal meanings means "sampling. In the mathematical cases above, they play the role of paradigmatic samples in which in and through the singular the universal can be grasped.

4. 3. 4 Axiomatic definition

Bibl.st.: A. Virieux-Reymond, *L'épistémologie*, PUF, 1966, 48/52 (*La méthode axiomatique*). G. Peano (1858/1932), one of the founders of logistics, defined the concept of positive integer as follows.

GG. The logical terms "class" (set), "member of a class" (instance) and "implication" (entailment: if, then); the number mathematical terms "number," "0," "1, 2 ..." instances of number), "a, b ... " (letter numbers) are "supposedly known" (phenomenon or given).

GV. Definition that establishes both content and scope (the latter deductively) of the concept of positive integer. The OPL occurs in the following sentences.

- **1.** The successor of a number. If a is a number, then a+ (understand: a+ 1), i.e., the successor of a, is also a number.

- **2.** Two indistinguishable numbers also have two indistinguishable successors. If a and b are numbers and a+ is the same as b+, then a is equal to b.

- **3.** Mathematical induction. If s is a class of which 0 is a member and every member of s has a successor within the class s, then every number is a member of s. Note that if a property is a characteristic of 0 as a member of the class s AND if that property is also a characteristic of the successor of 0, then it is a characteristic of all numbers in that class.

Or in other words, the characteristic in question is a common property of all instances of the term in question. - One generalizes starting from 0 and 0+ to all other members of the class (concept) S.

- **4.** The positive integer. If a is a number, then a+ (the successor of a) is not 0.

Abbreviated. 1. 0 is a number. 2. The successor of a number is a number. 3. Multiple numbers cannot have the same successor. 4. 0 is not the successor of any number. 5. Mathematical induction (see above).

System. Although the sentences - axioms - are mutually irreducible (and thus independent of each other, if not there is redundancy (redundancy)), yet they are valid only collectively and must be mutually consistent (contradiction-free). Only then do they form a logical system. These axioms are a definition such that the content, all the content and only all the content of the concept "positive integer" is distinguishable from the rest of all that is.

Magnitude. Since 0 is a number, the formation of tens, hundreds, etc. is possible within the system but since 0 is not the successor of any number, negative numbers - within the system, that is - are unthinkable ("non-existent"). The magnitude changes if we omit the sentence "If a is a number, then a+ is not 0" and replace it with "0 is the successor of -1 ", then - as it is said - the system weakens and negative numbers become 'conceivable' within that more comprehensive system which is then actually another system. The size to which the content refers is shown by the totality of all possible arithmetic operations which the axioms allow, and which constitute the infinite richness of them.

One sees that the system constituting the definition is a concept whose content is expressed in the sentences and whose extent is revealed by the operations (deductions) that are possible from the definition. Together with the definition, the set of all deductions forms an "axiomatic-deductive system."

4. 3. 5 Aristotelian axiomatic - deductive method

Bibl. st.: E.W. Beth, *The Philosophy of Mathematics from Parmenides to Bolzano*,

Antwerp/Nijmegen, 1944, 63vv. Steller treats Aristotle's concept of "axiomatic - deductive method" in the context of his notions of mathematics at that time. He calls this "Aristotelian theory of science" against which it should be noted that besides deductive science Aristotle also knew reductive science.

Definition of "deductive science". It includes as a concept definition what follows. 'W is symbol shortening for a system of sentences such that:

1. all sentences of W to a defined scope (area) of 'real' data (objects) strike;
2. all of W's sentences are "true.
3. if some sentences belong to W, any logical inference from those sentences also belongs to W;
4. there is a finite number of terms to designate such that:

a. the meaning of these terms needs no further explanation;

b. the meaning of all other terms appearing in W, using these terms alone can be described;

5. there is a finite number of sentences identifiable in W such that:

a. the truth of these sentences is evident;

b. all other sentences of W are logically deducible from these sentences. Beth's

assessment boils down to this:

- note 1. That interprets Platonic-Aristotelian "realism.
- note 3. This defines the deductive method.
- note 4b and 5b. That defines, Beth says, similarity and coherence, what Plato called "stoicheiosis" (elemental theory).

Criticism. This one boils down to this. 'Realism' is to be understood in the strictly ontological sense of "the belief that all that is not nothing but something, is 'real'." Thus the expression "ax + b = c" is not nothing but something and therefore ontologically something real. Stoicheiosis can be defined more broadly than just the theory concerning the "first axioms" of a deductive method. This is expounded elsewhere in this book (cfr. 9.2) as Plato's theory of order based on similarity and coherence. But admittedly: the application here is one case of this: the sentences of an axiomatic deductive account form a system of similarity and coherence.

- Re 4a and 5a. This is called "the evidential postulate." One can indeed argue about what in Aristotle's language "need no further explanation" and "being evident" mean. In this he is bound to be time-bound. But elsewhere (cf. 1.2.4) we discuss the misrepresentation by eristicians (especially Electra) of Aristotle's notion of obviousness. A more recent theory of axioms does specify more precisely what one means by "needing no further explanation" in that context. The whole question is: "Aristotle, if we interpret him as his works show him, would he reject these more recent precisions?". That he made no statements, for example, concerning the origin (induction, abstraction) of the axioms, only means that he, like every thinker, did not foresee, let alone answer, all the questions after him.

Conclusion. His definition of the axiomatic-deductive method, subject to precisions, is essentially valid.

4. 3. 6 The axiomatic deductive system interpreted ontologically.

Bibl. St.: St. Barker, *Philosophy of Mathematics*, Englewood Cliffs 5N.J.), 1964, 23f. (Terms.Axioms); - E. W. Beth, *The Philosophy of Mathematics*, Antw./Nijmeg., 1944, 63 vv. (The Aristotelian Theory of Science).- Summarizing these works and improving them if necessary, the structure of the system of judgments based on axioms and elaborating them deductively boils down to the following.

1. An axiomatic-deductive system includes:

a. a finite number of basic notions ("primitive terms") that are unproven presupposed but not chosen without sufficient (even if provisional) reason (as we saw in Peano's definition of the positive integer);

b. a finite number of basic theorems ("primitive theorems" or axioms, likewise unproven but not without at least a preliminary sufficient reason postulated. For example, Barker, o.c., 24 (Euclidian geometry) says that David Hilbert (1862/1943) postulated the concepts of "point / line / plane / incident / between / congruent" and E.V. Huntington postulated only "sphere / enclose in" as basic concepts for all Euclidean geometry.

2. From this, if the system "closes," all propositions that expose the scope of the conceptual contents must be derived strictly deductively provable.

Points 1 and 2 justify the name "axiomatic deductive."

Truth of such systems: - Aristotle, speaking of such axiomatic - deductive systems, argues that they contain objective - ontologically comprehensible - truth. This is often doubted by intellectuals who are not sufficiently familiar with ontological language. Behold:

1. The ancient Greek (alètheia in Greek) alètheia, unhiddenness, is first of all a purely phenomenological concept. Thus, those who engage in axiomatics and deduction from it start from data (phenomena, i.e., what shows itself directly, i.e., truth in the strictly phenomenological sense).

2. Even the most bizarre and fantastic mind-constructions, insofar as they are not contradictory in themselves, are "formae," realities, beingnesses, non-nobles and thus within strictly ontological language, "objective. Both mentioned properties of axiomatic-deductive systems together make them "objective reality" in its way, i.e.

reality in the ontological sense, show.

This explains why D. Van Dale, *Philosophical Foundations of Mathematics*, Assen / Amsterdam, 1978-4, can ask the very sensible question, "Do sets exist? (Existence question) and "What are sets?" (Essence Question). But that is precisely pure ontology, i.e. mathematical mind products.

4. 3. 7 Full evidence

In ancient Greek 'epicheirèma' (approach, basis of operation). Aristotle defines 'epicheirèma' as "short argument." By this he means a syllogism in which every preposition is provided with evidence. If we turn to this, it can be defined as follows: "A series of reasoning operations (basic concept), in an order that step by step includes all and preferably only all reasons (added concept) in such a way that a complete proof is provided (defined concept)."

Note: (1) The subterm "all and only all" in the above definition shows that it involves summative or Aristotelian induction. (2) A process frequent in mathematics and computing, namely the "algorithm," is one type of it. In the XII century, the computational rules (adopted from India) of the Islamic mathematician Al Chwarizmi were translated into Latin with the title "Algorismi de numero Indorum." The term "algorithm" dates from that time.

He also means "a purposeful series of logically sound thought operations." We give a few examples. They both happen to interpret a deductive proof.

Legal. M. T. Cicero (-106/-43), in his Pro Milone (Discourse t.v. Milo), develops a step-by-step proof and this is in the form of a syllogism.

VZ 1. For all cases, it is justifiable in conscience to kill an unjust assailant - o.g. legitimate self-defense - himself first. Proof. (a) The natural law (understand: the rules of conscience imparted with the general nature of man as a human being), (b) the positive (also "stellar") law (understand: the legislations introduced by human beings) justify such self-defense.

Note: Cicero thus posits an ethical-legal axiom or "principle" regarding morality and legality.

VZ 2. Well, Clodius, who threatened Milo, was such an unjust assailant. Evidence. (a) Clodius' criminal past ("his antecedents"), (b) his questionable escort, (c) the weapons found are evidence of his wrongdoing in this regard. Note: Milo's situation as unjustly attacked is a singular application of the universal axiom set forth in VZ 1. Immediately the deductive nature of Cicero's reasoning is clear.

Final sentence : So Milo was allowed to kill Clodius first.

Math. Bibl. st.: J. Anderson / H. Johnstone, Jr., *Natural Deduction (The Logical Basis of Axiom Systems)*, Belmont (Cal.), 1962,4.

To prove: $x((y + z) + w) = (xy + xz) + xw$.

The axioms already given include: $x (y + z) = xy + xz$.

1. By axiom: $x((y + z) + w) = x (y + z) + xw$.
2. By the same axiom: $x(y + z) + xw = (xy + xz) + xw$.

Which was provable.

The author : "A mathematical assertion is proved by exhibiting it as the consequence of assumptions."

Note: Immediately with this is given a minuscule specimen of what is called "axiomatic-deductive reasoning": on the basis of axioms one reasons from a given formula to a formula to be proved (demanded). From a purely logical point of view, between

Cicero's reasoning (on the basis of an axiom, he reasons about whether Milo acted conscientiously or not) and that of Anderson / Johnstone, Jr. (on the basis of an axiom, they reason about whether the requested formula is provable or not) do not differ substantially. In both cases one reasons step - by - step in a conclusive order, the "epicheirèma" mentioned by Aristotle, i.e. strictly logical approach.

4. 3. 8 Analysis (letter language)

Bibl.st.: O. Willmann, Geschichte des Idealismus, III (Der Idealismus der Neuzeit), Braunschweig, 1907-2, 48ff. Fr. Viète (Lat.: Vieta; 1540/1603) was a Platonist, familiar with the lemmatic-analytic method: one pretends that the GV (requested, sought, the unknown) was already GG (given, known) and introduces that already given, in the form of a lemma or "prolèpsis. In mathematics, for example, that lemma is denoted by 'x'

Number arithmetic. "Logistica numerosa". Before Viète, Western mathematics knew practically only numerical arithmetic. Thus e.g. "3+4 = 7".

Letter math. "Logistica speciosa". In his In artem analyticam isagoge (Introduction to Analysis), Viète worked with Platonic ideas, in Latin "species. This gives "ideative calculus." An idea is a universal set. Consequence: instead of working with singular or even private numbers, he worked with universal numbers. The following diagram clarifies the evolution.

COMMON LANGUAGE	FIGURE	LITTLE
The sum of two numbers	3+4=7	a+b=c
non-operative	operative	operative
universal	non-universal	universal

I.M. Bochenski, *Philosophical methods in modern science*, Utr./Antw., 1961, 55v. (Eidetic and operative sense), Ill.

(a) A sign has "eidetic" meaning if one knows the reality of it to which it refers (the semantic interpretation is known).

(b). A sign has "operative" meaning only if one knows how to deal with it without thinking of its eidetic or semantic meaning. "We do not know what the sign means, but we do know how to operate with it." (O. c., 55).

The latter is clearly the case with number language (not - universal) but overwhelmingly the case with letter language (universal) because letters are "fillable" by - in principle - any number. Which, conversely, is not the case.

If the eidetic meaning is known - e.g. 3 + 4-, then an operative meaning is immediately available (e.g. 3 + 4 = 7). Not vice versa: one can assign an operative meaning to a sign without any semantic meaning (e.g. a + b = c).

Logical syntax. - Thus Viète founded a syntactic (= operative mathematics) with his letters as lemmas. Analysis is thus the elaboration of what one can do with those lemmas (empty shells) concerning mathematical operations - logically justified. Thus arose e.g. analytic geometry". The name testifies to the lemmatic analytic method.

Those who are purely operative work with lemmas of a special type: the general content (e.g., a as a known number) is known, but as an empty shell waiting to be filled in (e.g., a as 3).

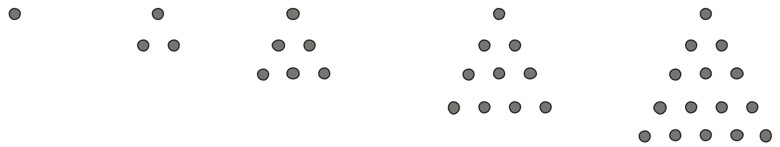
Viète's process is twice Platonic.

1. The process is ideational, because he works with ideas as empty shells with universal scope (e.g., a represents all possible numbers as fillings) and thus with sets.

2. The ideas are ipso facto lemmas, usable in the course of an analysis precisely in virtue of the fillings and corresponding operations (which shows the operative character of mathematical ideas). - Viète himself says: "Analysis is working with the requested ('queaesiteria') as if the given ('concescum') was such that by means of the inferences therefrom the requested itself is exposed."

Note: The rule of three shows this: " If 100% (the universal idea) is 25 and if 1% (the singular instance) is 25/100 then 10% is 10.25/100". The requested itself is the result i.e. 10.25/100; the lemma is 10% drawn via 100% and 1% exposed. It also appears that analysis consists in situating the requested in the form of the lemma (the as if given; here 100%) in a network of relations, here the structure of the rule of three.

The triangular numbers of the Pythagoreans: These are obtained by adding successive natural numbers. If they are represented in spatial structures, they form isosceles triangles.



1 3 5 10 15

The following structure each includes the previous one plus a new base added to it. These triangle numbers answer to Heath's formula: $N = n(n+1)/2$ where N represents the total number of units, and n represents the number of units making up the base of the triangle.

This formula is the idea as a lemma for the visualizing models of the Pythagoreans with their triangular numbers.

Extensions. Willmann, o.c., 48f Viète's revolution was elaborated.

1. Functional Theory. The unknown ('lemma') a can be replaced by x, i.e., a variable (variable) unknown. Thus: $x = y+z$, where x is the dependent variable and y and z are independent variable unknowns such that x is 'function' of y+z.

2. Analytic geometry. The name 'analytic' still recalls Plato's 'analusis'! R. Descartes (Géométrie (1637) and P. Fermat (1601/1665) founded 'analytic' geometry just about simultaneously in Viète's wake. Thus the formula "$r^2 = x^2+y^2$". Where r is the 'radius' of the circle, drawn against the background of Cartesian coordinates (two lines intersecting rectangularly, the X-axis and the Y-axis). Plotted circles are indeed "illustrative models" but they are little or not even operative. The letter figures in their variable form are a general formula summarizing all possible illustrative circles.

3. Infinitesimal arithmetic. The lead-up to this is found in Nicholas of Cusa (1401/1464) where he talks about the evolution of quantities (under Pythagorean influence). G.W. Leibniz (in 1682) founds infinitesimal mathematics (working with differentials and integrals).

Behold the transition from the "eidetic" treatment of quantity and to its "operative" treatment. As Bochenski says: when, in dealing with operative formulas, we are "only" apply syntactic (character-binding) rules, then a "logical syntax," an interconnection of characters on a logical basis, works perfectly.

Logistics will take this much further, of course. There logic becomes a "calculus," an arithmetic, with "empty" but "fillable" symbols. An end point of the Platonic lemmatic - analytic method.

4. 3. 9 Logical independence of mathematics

Bibl. st.: Ch. Lahr, *Cours*, 564/566 (*Mathématiques modernes et géométries non - euclidiennes*). A. Virieux-Reymond, L'épistémologie, PUF, 1966,48/52 (La méthode axiomatique).

Logical independence. A model. In traditional arithmetic, one defines a fraction by starting from measurable data: "Divide an apple in half" or "Divide the number 10 by 2." 'Modern' it becomes as follows: "A set of two numbers, a and b, if suitable in the following configuration a/b, is a fraction number". One of the properties is expressed as follows: "Two fractional numbers, a/b and c/d if ad = bc, are equal". From such definitions a theory of the fractional numbers is deducible without recourse to sense. That 'without' is "the logical independence" (from sense intuition) of 'modern' mathematics, as it was constructed during the XIXth century. It would retain its 'value' even if measurable quantities never existed. It draws its 'justification' from its contradiction-free system character.

One begins with pure symbols as a language in which basic concepts and basic axioms are formulated (expressed in formulas) from which one deduces - independent of any sense-perception, according to rules of deduction - propositions. This is called "formalization" and allows "calculus" (logical arithmetic) within an axiomatic-deductive system.

Non-euclidean geometry. Euclid's definition of a straight line is logically dependent on the sensory intuitions we have of a "straight line." However, if one proceeds independently of any sensory intuition, one can add to the Euclidean definition the axiom of Bernhard Riemann (1826/1866), namely, "Through a point outside a straight line one cannot draw a parallel straight line." That creates a non-euclidean space mathematics. Or one can add the axiom of Nikolaï Lobachevsky (1792/1865), namely, "Through a point outside a line one can draw an infinite number of parallel lines." The logical validity of the space mathematics of Riemann and Lobatshevsky is equivalent to that of Euclid.

The reality character of formalized number and space mathematics depends on how one defines 'reality'. If 'real' e.g. means 'existing outside the human mind', then constructions like the formalized mathematics are 'unreal'. If, however, one defines 'real' ontologically, then 'real' is "all that however is not nothing but something." The constructions of the human mind - from pure science fiction or utopia to logistics or formalized mathematics - are "not nothing" and thus ontologically real. Logical independence does not yet imply that they get outside the realm of well-understood - and not confused with non-ontological conceptions - ontology. Too bad: a lot of even intellectually educated people confuse ontological language with what they think they know about it! As an aside, this book has a brief exposition of what ontology (theory of reality) is to clear up just such confusions.

4. 3. 10. This chapter summarized:

Mathematics is applied logic, but also a logically coherent system of objective sentences. For some it is a construction of the mind, for others a reality in itself. Still others see it as a realization of Platonic ideas.

Mathematics can be defined as the science of quantity and space, and of the system of symbols connecting quantity and space."

According to mathematicians, mathematics is the only science that provides definitive and irrevocable proofs, while the other subject sciences provide situational tests.

One isosceles triangle can be a model for all other isosceles triangles. From that one triangle, one can show that the opposite angles are necessarily equal. Thus, amplificatory induction is logically justified.

One can determine the sum of a number of consecutive odd numbers, starting with 1, through sampling and discover the rule in them. Thanks to algebraization, from this summative induction, one can find the formula for all cases and thus arrive at the amplificative induction.

G. Peano, one of the founders of logistics, defines the concept of a positive integer from a number of premises, such that its content and scope are fixed. Definition and deductions together form an axiomatic deductive system. The sentences of an axiomatic-deductive account form a system of similarity and consistency. Provided clarifications, the definition of Aristotle's axiomatic - deductive method remains valid.

Ontologically, an axiomatic deductive system consists of a finite number of unproven basic concepts and a finite number of basic propositions. From these, all theorems that expose the scope of the concept contents must be deductively derived.

Aristotle argues that they contain ontologically objective truth.

'Epicheirèma' can be defined as a series of successive reasoning operations, covering all and preferably only all reasons such that a complete proof is provided . The 'algorithm', is one type of it.

The lemmatic - analytic method pretends that the GV was already GG and introduces that already given, in the form of a lemma. Viète transformed number arithmetic into letter arithmetic, which allowed him to work operatively with universal numbers. Viète's revolution sees further elaboration in function theory, analytic geometry and infinitesimal calculus working with differentials and integrals. Logistics will develop this further.

The logical independence of mathematics consists in the fact that from definitions a theory is deducible without having to appeal to sense-perception. It draws its 'justification' from its contradiction-free system character. This is called 'formalization' and allows 'calculus' (logical arithmetic) within an axiomatic - deductive system.

If one proceeds independently of any sense intuition, then one cannot create - euclidean forms of geometry. The reality character depends on the definition - ontological or not - one wants to give to the concept of reality.